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Working Paper

World Class Standards For Preparing Teachers of Mathematics

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Introduction

International comparative studies in education provide a fresh perspective on K-12 education policy by enabling countries to learn from each other's approaches. The recently conducted Teacher Education and Development Study – Mathematics (TEDS-M) provided us with a world-wide lens to examine some of our practices related to the preparation of U.S. teachers of mathematics for elementary and middle school students. More specifically, by looking at the top performing teacher preparation programs internationally we found a common set of learning experiences. These empirically derived standards can be used to examine the quality of the preparation of future U.S. teachers in various university and college programs. Analyses based on the TEDS and TEDS follow-up studies indicate considerable variation in the rigor of U.S. teacher preparation programs (especially for middle school), and that the coursework of future teachers can have a significant impact on their preparation to teach mathematics.

TEDS-M

Close to 23,000 potential future teachers of mathematics enrolled in the final year of teacher preparation from nearly 900 programs participated in the Teacher Education and Development Study – Mathematics (TEDS-M) conducted in 17 different countries. In the U.S. data were collected in two consecutive years: 2008 at public colleges and universities and 2009 at private colleges and universities (Center for Research in Mathematics and Science Education, 2010). Follow-up studies were also conducted on teachers after they had already begun teaching (see below).

This ground-breaking international study of higher education with outcome measures focused on two different future teacher populations: 1) those who were prepared to teach in the primary grades, typically grades 1-5; and 2) those prepared to teach mathematics at the lower secondary level (i.e. middle school), usually grades 6-8. (see Tatto et al, 2012). In most countries primary teachers are not specialists; mathematics is only one of several subjects they teach. Middle school teachers, however, typically are specialists who are prepared to teach either only mathematics or mathematics along with one other subject. TEDS-M assessed the future teachers' knowledge about mathematics related to what they will be teaching (MCK) and their knowledge related to the teaching of mathematics (PCK) as well as the learning experiences they had encountered during their teacher preparation. The PCK measure sought to capture the instructionally embedded mathematical knowledge thought to be critical for the professional work of teachers (Ball & Bass, 2003; Ball, Hill, & Bass, 2005).

Identifying an International Benchmark for Middle School Teachers

To create an empirically derived benchmark we identified the top ten percent of programs across all 17 countries based on the mean MCK score for future teachers in each program. We then identified the courses that the vast majority of these future teachers reported having taken. The programs making up the top ten percent were essentially the same whether based on the MCK or the PCK score. This may be explained in part by the relatively high correlation at the program level between these two scores (.93). The top ten percent of middle school programs included 39 programs from four countries: Poland (1), the Russian Federation (15), Taiwan (17), and the U.S. (6).

To qualify as part of an international benchmark for teacher preparation programs, courses had to be taken by 80 percent of future teachers within at least 90 percent of the top performing programs. This procedure identified 9 courses which we refer to as "core requirements. " The 90% of programs criterion ensured that the benchmark was truly international, widely practiced, and not unduly influenced by any one program, especially those with large numbers of future teachers (see Schmidt and Cogan, 2013 for more detail). As a part of the benchmark we also identified a set of electives which were taken by most of the future teachers but were not as widespread across the programs as the core requirements. Electives were identified as 80% of future teachers within 80 percent of top performing programs. The resulting course benchmarks for middle school are displayed in Table 1.

Table 1. Empirical requirements and electives identified from the top-performing international middle school programs

Requirements	Electives
University Mathematics	**University Mathematics**
Beginning Calculus	Abstract Algebra
Calculus	Analytic Geometry
Multivariate Calculus	Axiomatic Geometry
Differential Equations	Number Theory
Linear Algebra	Set Theory
Probability	
Math Education	**Math Education**
Math Instruction	Math Standards
Observing/Analyzing Math Teaching	
School Mathematics	**School Mathematics**
Functions/Equations	Geometry
	Numbers
	Statistics

It is rather remarkable to find so many courses as part of the middle school benchmark given that it was derived from programs in four different countries on three continents. The set of nine core courses included six university mathematics courses (beginning calculus, calculus, linear algebra, probability, differential equations, and multivariate calculus); two math education courses (mathematics instruction; and observation, analysis and reflection on mathematics teaching); and one school mathematics topic (functions, relations, and equations). The math education course (observation, analysis and reflection of mathematics teaching) was experienced by the smallest share of future teachers across all 39 top-performing programs, 92 percent. By contrast, each of the three university mathematics courses (beginning calculus, calculus, and linear algebra) were taken by 99 percent or more of the future teachers. Among this set of 9 courses there is a strong emphasis on the content of mathematics with only two focusing directly on mathematics instruction. A similar emphasis was found among the electives, which included five additional math courses, three school math courses and a pedagogy course focusing on the country's mathematics standards.

The preparation benchmarks provide an empirical basis for what may be considered a world-class preparation to prepare middle school mathematics teachers. Table 2 shows what percentage of U.S. future teachers attained this benchmark during their preparation programs. The percentages included in this table are for those who took at least 8 of the 9 core courses. These future teachers also took a number of popular elective courses. By way of comparison, U.S. results are compared with those of the Russian Federation and Taiwan, whose MCK scores were significantly higher than the U.S. (Center for Research in Mathematics and Science Education, 2010). In the U.S. we also divided the sampled programs into subgroups: the top 25% and the bottom 25% of programs as defined by mean MCK scores.

Across all U.S. middle school teacher preparation programs, only around one-third (31 percent) of future teachers reached the international benchmark. In the two comparison countries, 95 percent or more reached this benchmark. Additionally, while over 75% of all future teachers in Taiwan and Russia took all 9 benchmark courses, less than 15% of U.S. teachers did so. Differences among groups of U.S. lower secondary programs were striking. In U.S. teacher preparation programs that were in the top 25% in the world, nearly 70 percent of the future teachers in U.S. programs reached the world-class benchmark. In the top quarter of programs *in the United States,* this percentage increased to nearly 77 percent. By contrast, only 11 percent of the future teachers trained in programs with lower average MCK scores (in the bottom 25% of the U.S. distribution) reached the benchmark criterion.

Not only is the overall percentage of U.S. future teachers meeting the international benchmark quite low (less than a third), and especially in comparison with Russia and Taiwan, but this overall figure masks the tremendous variation across programs. The difference in mathematics course-taking between those programs at the top of the U.S.

distribution and those at the bottom is striking. The fact that only 11 percent of future teachers in the lowest-MCK programs meet the standard is made more salient by our estimate that these schools prepare **60 percent** of all the nation's future middle school teachers.

Table 2. Percent of future middle school teachers reporting they had experienced at least 8 of the 9 core international benchmark courses.

	U.S.A.	*Russian Federation*	*Taiwan*
Future Lower Secondary Teachers In			
All Programs in Country	31.5%	97.2%	94.5%
U.S. Top 25% of Programs	76.9%		
U.S. Bottom 25% of Programs	11.3%		

Table 3 provides greater detail on U.S. future middle school teachers course-taking, related to the international benchmark. The least likely benchmark course to be taken by these U.S. future teachers was differential equations. Among future teachers in the top 25% of U.S. programs, only 70 percent reported having had this course. At the bottom quarter of U.S. programs, less than 30 percent of future teachers reported having taken differential equations. An even more striking difference between the top and bottom quarter of U.S. programs, however, is seen for multivariate calculus: nearly 95 percent of future teachers took multivariate calculus in the higher-performing programs as compared with only *11 percent* in the bottom quarter of programs. Most future teachers in the bottom quarter of U.S. programs reported taking the university mathematics probability course (80%). However, relatively few of these teachers reported having taken most of the other benchmark courses and hardly any indicated they had come close to the international world-class benchmark. Again, this is particularly disturbing given that enrollment data from the lowest quarter of programs suggest they produce approximately three-fifths of all future middle school mathematics teachers in the U.S.

Table 3. Percent of future middle school teachers in each type of program reporting they had experienced each middle school international benchmark course.

	U.S.A.			*Russian Federation*	*Taiwan*
	All Programs	***Top 25%***	***Bottom 25%***	***All Programs***	***All Programs***
International Benchmark					
University Mathematics					
Beginning Calculus	69.1%	97.7%	51.6%	99.5%	100%
Calculus	58.3%	95.8%	34.2%	99.7%	100%
Differential Equations	43.9%	70.2%	29.6%	99.3%	93.5%
Linear Algebra	67.6%	98.1%	52.0%	99.3%	100%
Multivariate Calculus	40.4%	94.8%	11.4%	96.1%	93.9%
Probability	84.9%	91.3%	80.0%	99.2%	99.2%
Math Education					
Math Instruction	75.9%	85.7%	74.6%	96.6%	95.1%
Observing Math Teaching	76.2%	84.2%	74.8%	93.1%	88.2%
School Mathematics					
Functions	80.6%	84.7%	77.5%	99.2%	97.6%

Identifying an International Benchmark for Primary School Teachers

The analysis described above was replicated in order to identify an international benchmark for teachers prepared to teach in primary schools (grades 1-5). For primary programs, the number of countries and the specific programs in those countries varied more when based on the MCK or PCK scores. The top ten percent of MCK-based programs came from seven countries: Norway (1), Poland (17), the Russian Federation (11), Switzerland (1), Taiwan (10), Thailand (5), and the U.S. (4). Based on PCK, the top ten percent programs came from eight countries: Norway (4), Poland (11), the Russian Federation (3), Singapore (1), Switzerland (2), Taiwan (10), Thailand (1), and the U.S. (17). A core of 29 programs were in the top ten percent for both MCK and PCK. These came from six countries: Norway (1), Poland (10), the Russian Federation (3), Taiwan (10), Thailand (1), and the U.S. (4).

For primary programs, the MCK-based requirements were a subset of those based on PCK. Consequently the primary benchmark discussed represents the union of the MCK- and PCK-based requirements (unlike the secondary benchmark, which was based only on MCK scores). The courses identified are presented in Table 4.

Table 4. Empirical requirements and electives identified from the international top performing primary programs

Requirements	Electives
University Mathematics	**University Mathematics**
Number Theory	Analytic Geometry
Probability	Axiomatic Geometry
Math Education	**Math Education**
Math Instruction	Math Standards
	Observing Math Teaching
School Mathematics	**School Mathematics**
Measurement	Functions
Numbers	Geometry

The emphasis in the set of international benchmark primary courses suggests a slightly different focus for primary teacher mathematics preparation. Although the vast majority of the required courses are mathematics oriented – four out of five (80%) – the two university mathematics courses, number theory and probability, and the K-12 school mathematics courses, measurement and numbers, suggest a less rigorous or advanced formal mathematics emphasis than that for middle school. This finding is not particularly surprising, given that mathematics is not the principal emphasis of primary teacher preparation or instruction.

Table 5. Percent of future primary teachers reporting they had experienced at least 4 of the 5 international benchmark courses.

	U.S.A.	***Russian Federation***	***Taiwan***
Future Primary Teachers In			
All Programs in Country	84.5%	95.3%	91.1%
U.S. Top 25% of Programs	*85.8%*		
U.S. Bottom 25% of Programs	*86.5%*		

As with with Table 3, Table 5 compares the proportion of U.S. future primary teachers meeting the international primary benchmark compared with Taiwan and the Russian Federation. For U.S. primary future teachers, the percent reaching the world-class benchmark was rather constant whether the subgroup of programs came from those in the top or the bottom 25% of U.S. programs, or represented the entire U.S. primary program sample as a whole. Although this percentage was rather high, about 85 percent, it was not as high as the corresponding percentages in Taiwan and the Russian Federation. A majority (56%) of U.S. future teachers took all 5 benchmark courses. Clearly more future elementary teachers meet the relevant international benchmark than was the case for the middle school teachers, a distinction we will discuss further below.

The Long Term Influence of Teacher Preparation Programs: Results from the U.S. TEDS Follow-Up Survey

Building on the work of the original TEDS-M, the Center for the Study of Curriculum conducted follow-up research on graduates from U.S. teacher preparation programs that participated in the original TEDS. These surveys probed respondents about their experiences as K-8 teachers, incorporating questions covering a broad range of topics, including how well prepared they felt to teach a list of mathematics topics after two years of teaching. Linking this information to the results of the original TEDS-M enabled an exploration of the long-term impact of teacher preparation programs and the differences among these programs. We also examined what sorts of schools new teachers were employed by, and how the characteristics of these schools were related to how teachers were prepared.

Approximately 2300 of those involved in the original TEDS survey participated in the 2011 follow-up survey, about a 45% response rate. The 2011 follow-up was conducted two years after the original TEDS study. Analyses of varying demographics suggested that respondents were roughly similar to those taking part in the original TEDS, although participants in the follow-up survey had slightly better test scores.

Analyses of the follow-up survey results yielded two important insights about U.S. teacher preparation to teach mathematics. First, the survey revealed that the least well-prepared new teachers were more likely to be hired by the highest-poverty schools (see Table 6). TEDS follow-up responses about which school respondents were employed were merged with National Center of Education Statistics school demographic data. We found that the primary school teachers who scored in the top 25% of the U.S. sample on the mathematics content test (MCK) were more likely to be employed by a low-poverty school (defined by the NCES as 25% or fewer of all students eligible for free and reduced meals) than a high-poverty school (75% free and reduced meal eligibility). This relationship was statistically significant for primary school teachers, but not middle school teachers. Similarly, using the international benchmarks of primary (IPB) and secondary (ISB) teacher preparation course-taking discussed above, the analyses indicated that those future teachers who took the most demanding courses were more likely to be teaching at low-poverty schools than were less well-prepared teachers. Both of these associations were statistically significant.

Table 6. Teacher Employment by Preparation and School Poverty

Secondary	School Poverty	Lowest MCK	Highest MCK	Low ISB	High ISB
	0-25%	37%	40%	32%	44%
	75-100%	16%	13%	23%	12%
Primary	School Poverty	Lowest MCK	Highest MCK	Low IPB	High IPB
	0-25%	21%	44%	37%	36%
	75-100%	29%	16%	27%	13%

Second, we explored the relationship between teacher course-taking in their teacher preparation programs and how well prepared they were to teach the mathematics topics *after they had begun teaching*. Teachers were asked to evaluate their academic preparation to teach a set of mathematics topics on a (1-4) four point scale. Responses were averaged across topics, with higher scores representing more confidence to teach mathematics topics appropriate to their grade level. Regression analyses demonstrated that teachers whose course-taking more directly matched the international benchmark indicated they were better prepared to teach mathematics topics after they had begun teaching (see Table 7).

Course-taking had a statistically significant relationship to how well prepared they were even after controlling for a teacher's SAT or ACT score, their mathematical and pedagogical content knowledge, and whether they went to a public or private university. In fact, for middle-school teachers course-taking was a much stronger predictor of academic preparation than teacher knowledge (measured by MCK scores). The results suggested that future teachers who had taken all of the core courses defined by the international benchmark would be 25% more prepared than those who had taken none of the courses at least as reported by the teachers (i.e. an average of 1 point higher average on the 4 point scale)[1].

[1] Multi-level analysis confirmed these results.

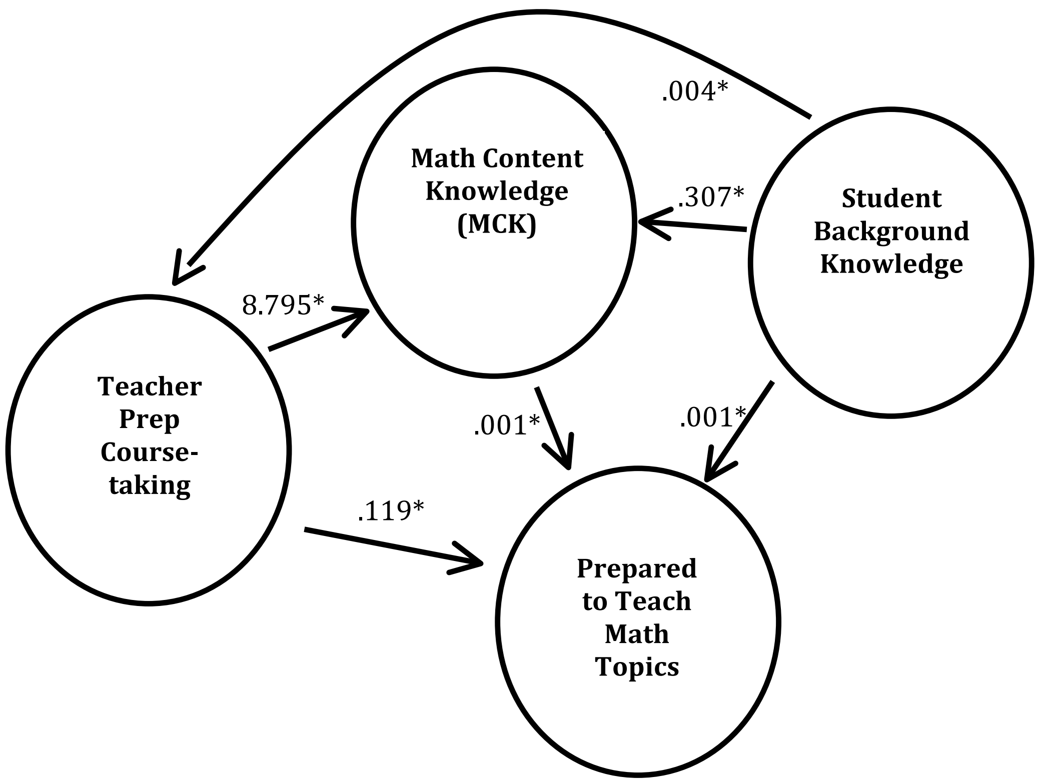
Table 7. Teacher Course-taking & Self-Reported Academic Preparation to Teach Mathematics Topics

	Prepared to Teach Middle School Math Topics	Prepared to Teach Elementary Math Topics
Secondary Course-taking	.119*	
Primary Course-taking		.135*
SAT/ACT	.001*	.001*
MCK	.001*	.002*
PCK	.001	-.001
Public	.114	.020
Primary Prep.	-.429*	-.116*
Constant	.520*	1.721*
adjusted r-square	.269*	.059*
N	421	655

*statistically significant at the .05 level

Course-taking was also related to individual MCK scores at the secondary and primary level. Our analysis therefore suggests that the course-taking patterns of future teachers have both a direct relationship with teacher confidence to teach mathematics topics as well as indirect relationship through its association with teacher knowledge of mathematics. Figure 1 presents the results of a path analysis of the factors influencing academic preparation to teach mathematics for middle school teachers, demonstrating that course-taking has a substantial total effect on teacher capacity. The relationships are similar for primary school teachers, although the predictive power of the model was weaker. Although we cannot say with certainty that this teacher confidence in teaching mathematics is translated into student learning, it nevertheless is an encouraging sign that teacher preparation can influence the long-term trajectory of teacher instruction.

Figure 1. Model Relating Teacher Preparation, Mathematics Content Knowledge, Student Background Knowledge, and Academic Preparation to Teach Middle School Mathematics.



Discussion

There are three principal conclusions to be drawn from our analysis of the TEDS-M and TEDS follow-up surveys. First, **there is a significant relationship between what teachers study in their teacher preparation programs and self-reported preparation to teach mathematics.** Previous research has provided strong evidence that opportunity to learn (OTL) – content coverage – has a strong relationship to K-12 educational outcomes. The research presented here suggests that course-taking is also a key factor in teacher preparation (examined in *Why Schools Matter,* Schmidt et al. 2001) Analysis of the course-taking patterns of the top-performing teacher preparation programs across 17 countries revealed a reasonable consensus about what courses are appropriate for preparing future teachers. For example, more rigorous mathematics content, such as university-level mathematics at the middle school level, characterized the best programs.

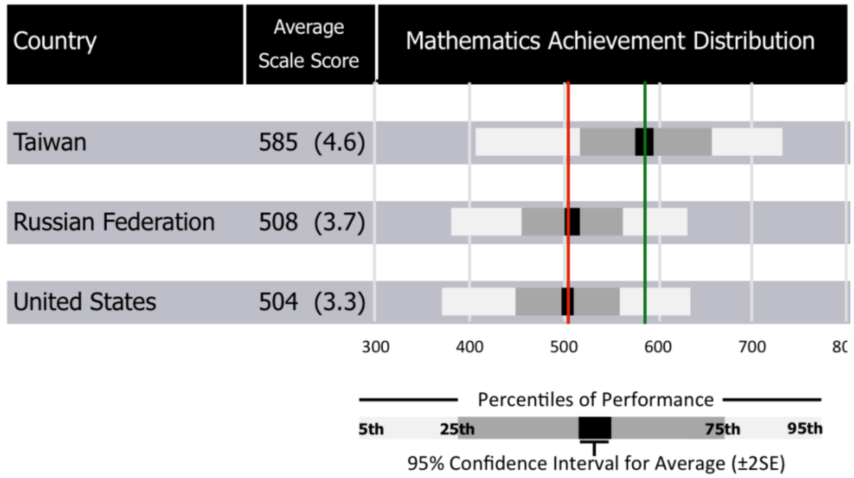
Results from the TEDS follow-up studies indicated that these international benchmarks are related to how well prepared teachers say they are after teaching mathematics for two years. Teachers whose course-taking reflected the international benchmarks demonstrated much greater confidence in teaching mathematics topics. The association between teachers' self-reported ability to teach mathematics and their previous course-taking is quite robust, even after accounting for the student's background and knowledge. A richer grounding in mathematics content could likely pay tremendous dividends in terms of student learning.

Second, **there are reasons to be concerned about the inadequate mathematics knowledge of U.S. teachers, both at the primary and middle-school level.** Although some U.S. teacher preparation programs are among the best of the world, the reality is that far too many teachers do not receive adequate mathematics training before they enter the classroom. This is a particular problem in middle school, as roughly three-fifths of such future math teachers graduate from the bottom quarter of teacher preparation programs in the U.S. This is especially disconcerting given the recent adoption of the Common Core State Standards by over forty states. In addition, our analysis reveals that the least-prepared teachers are more likely to be hired by the poorest, most-disadvantaged schools, exacerbating educational inequality. Further, despite the fact that the international primary benchmark only constitutes five courses, only a little over half of future U.S. primary teachers reported taking them.

Finally, **the TEDS studies underscore the critical difference between elementary and middle-school teacher preparation in mathematics**, a distinction that should be kept clearly in the forefront of researchers and policymakers. Although the basic relationship between teacher course-taking and confidence to teach mathematics is quite similar for primary and middle-school teachers, the overall impact is more muted in the case of primary teachers. Likewise, while the proportion of teachers reaching the benchmark varies wildly across different middle school programs (and countries), there is much less variation at the primary level. The differences between primary and middle-school preparation should be no surprise. Because they are math specialists, mathematics preparation is the dominant focus of future middle school math teachers. Primary school teachers, on the other hand, are expected to be proficient in teaching many subjects, and to do so at a more basic level. The fundamental difference in the content and grade-level focus of primary and middle-school mathematics teachers means that the expectations of teachers and the thrust of policy must be just as different.

For example, we have argued previously that because the U.S. population generally does less well on international K-12 mathematics assessments, American teacher preparation programs are drawing from a weaker pool of future mathematics teachers. To recruit future teachers with the same mathematics knowledge as the average person from Taiwan, the U.S. would have to recruit from the top quarter of U.S. eighth graders (see Figure 2). This is a particular issue in primary teacher preparation, as selection effects appear to play a greater role in influencing primary teacher's ability to teach mathematics. This may be due in part to the fact that mathematics makes up only a portion of the content teachers are expected to prepare for. As a consequence, improving the mathematics preparation of primary teachers represents a considerable challenge, one that requires a great deal of careful study.

Figure 2. TIMSS 2003 Eighth Grade Mathematics Achievement Distributions.



Source. Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. (2004). TIMSS 2003 International Mathematics Report: Findings From IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grade (pp. 465). Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College. Figure reproduced from CRMSE 2010.

Secondary teacher preparation is a different story, in that course-taking patterns within teacher preparation programs have a stronger relationship with mathematics knowledge and preparation to teach mathematics. The fact that the majority of future U.S. secondary teachers graduate from the weakest programs raises concerns about the quality of middle school mathematics teaching in the U.S. However, the results also point to a possible remedy: the international benchmark for secondary teacher course-taking suggests that improvements in the course requirements of teacher preparation programs might improve the performance of U.S. middle school mathematics teachers, and ultimately that of their students as well.

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